EVALUATION OF HYPERELASTIC MODELS FOR TIBIAL ARTICULAR CARTILAGE UNDER HIGH STRAIN RATE LOADING

Jessica M. Deneweth, Shannon M. Pomeroy, Scott G. McLean and Ellen M. Arruda
University of Michigan, Ann Arbor, MI, USA; email: jmden@umich.edu

INTRODUCTION

Modeling articular cartilage (AC) poses a challenge due to the tissue's complex physiology and its equally complex non-linear viscoelastic loading response [1, 2]. Under the relative short loading times associated with walking, the mechanical response of knee joint AC is primarily driven by the collagen network of the superficial zone [3]. Recent work [4] has suggested that a hyperelastic statistical chain model, which treats a material as a network of flexible chains, could model tibial AC compressed at 0.1 and 0.025 strain/s. Such a model is intriguing because its structure is reminiscent of the AC collagen network, it handles finite deformations well, and its mechanistic nature has the potential to reveal insight into factors driving AC mechanics. However, it is unclear how such a model would behave under higher strain rates associated with normal human activity or how well such a model can replicate the non-uniform mechanical properties of tibial AC [5].

The purpose of this study, therefore, was two-fold: 1) to determine which, if any, of three popular statistical mechanics models could successfully simulate the unconfined axial compression behavior of tibial cartilage tested at 1 strain/sec, and 2) to determine the extent to which the model that best fit the data could capture the regional mechanical variations evident within the tissue [5].

METHODS

Three statistical chain formulations were investigated: the isotropic eight-chain network model (Figure 1) with freely jointed chains (FJC) [6], the isotropic eight-chain network with MacKintosh chains (MAC) [7], and a novel transversely isotropic eight-chain network with freely jointed chains (TI). The TI model was developed by modifying an orthotropic eight-chain network model [8] such that the stiffness parameters (b, c) and stretches in the plane perpendicular to compression were assumed equal (Figure 2). This simplification represented a first-order approximation of the anisotropy of the AC superficial zone, in which the collagen fibrils are primarily aligned parallel to the AC surface.

Each model was used to simulate 100% strain/s unconfined compression of healthy human proximal tibial cartilage [5] using a non-linear least-squares optimization algorithm implemented in Matlab (Mathworks, Natick, MA). The goodness of fit of each model was evaluated from its R^2 value.

The parameters of the model with the highest R^2 were then analyzed for regional dependence. Regional heterogeneity was evaluated by dividing the medial and lateral tibial plateaus into four regions: not covered by meniscus (I) and the anterior (II), exterior (III), and posterior (IV) thirds of the meniscus-covered area [5] (Figure 4 inset). These values were submitted to a repeated-measures mixed model analysis of variance to test for the effects of plateau side (n=2), region (n=4), and the interaction of side*region (n=8). Bonferroni-adjusted pairwise comparisons and Cohen's d were calculated for all main effects.

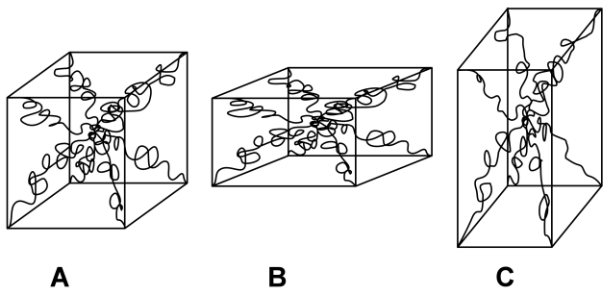

Figure 1. Schematic of the isotropic eight-chain network model [6]. Eight chains originate at the center of the unit cube of material and extend to its corners (A). The chains stretch and rotate as the cube is deformed, which gives rise to tissue stress (B-C).

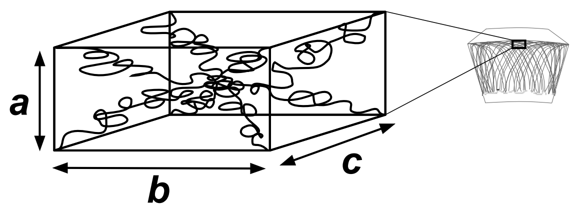

Figure 2. Schematic of the transversely isotropic eight-chain network. The dimensions in the plane parallel to the AC surface (b, c) are equal and different from the third dimension (a). The ratio of a:b represents the relative anisotropy of the material.

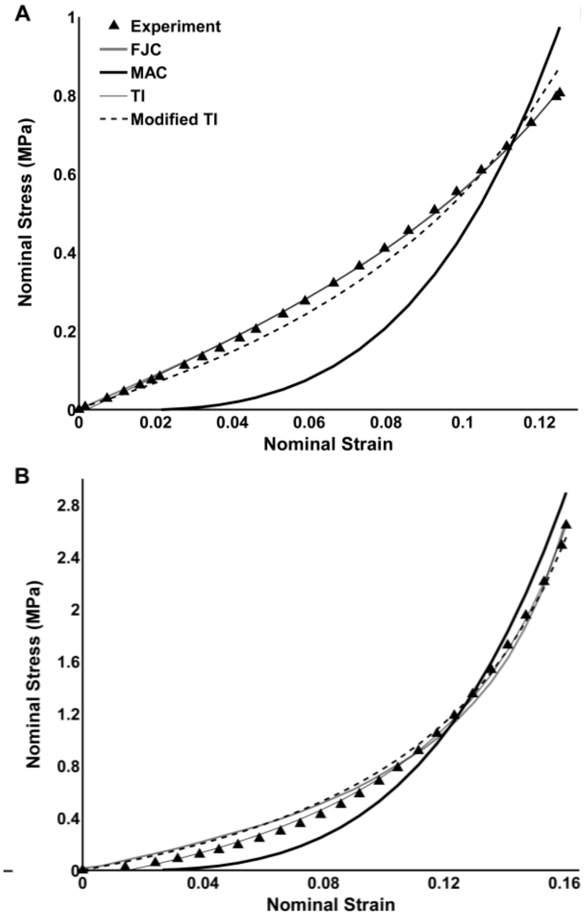

Figure 3. Representative simulation results for the FJC, MAC, TI, and modified TI models for (A) nearly linear and (B) non-linear stress-strain responses. The TI model provided the best fit to the experimental data.

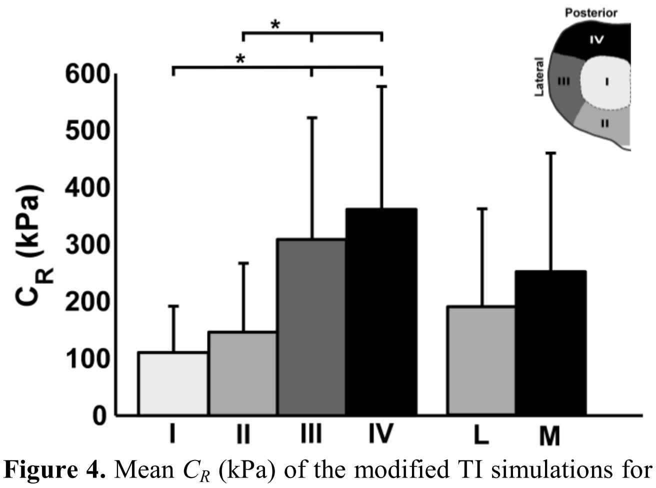

Figure 4. Mean C_R (kPa) of the modified TI simulations for the four regions (I – IV) and the lateral (L) and medial (M) plateaus. Bars represent one standard deviation. Asterisks (*) denote statistically significant differences ($p< 0.05$). The regional differences in C_R mirrored the regional pattern of tibial AC elastic moduli [5].

RESULTS AND DISCUSSION

The average R^2 was 0.995, 0.900, and 0.999 for the FJC, MAC, and TI models, respectively. TI was particularly superior to FJC in the low-strain region of non-linear data (Figure 3). The TI model, therefore, was used for the second part of the study.

Before performing the regional analysis, the TI model was modified based on two key assumptions: 1) AC anisotropy was assumed to be identical across the tibial plateau and 2) AC was assumed to be incompressible. This reduced the unknown parameters from four (C_R, a, b, J) to one (C_R). The modified TI model fit the experimental data with a mean R^2 of 0.983 (Figure 3).

Significant pairwise differences ($p < 0.05$) in C_R were determined between Regions I and III, I and IV, II and III, and II and IV (Figure 4). No statistically significant difference was evident, however, between the medial and lateral plateaus ($p = 0.069$). The mean differences between each pair of regions exhibited moderate to strong effect sizes except for I-II and III-IV on the lateral plateau and III-IV on the medial plateau. The effect size between the medial and lateral plateaus was small (0.32).

CONCLUSIONS

A new transversely isotropic constitutive relation has been developed and found to demonstrate excellent ability to model the mechanical response of tibial AC to high strain rate unconfined compression. When this model was simplified using well-accepted assumptions for AC, it retained a superb fit to the data. Furthermore, it demonstrated a regional dependence of its one unknown parameter, C_R, that was identical to the regional pattern of the elastic tangent modulus demonstrated experimentally [5]. This pattern also reflects the underlying collagen density of the tissue, suggesting that the model has the ability to reveal additional insight into cartilage behavior.

The simplicity of this modified transversely isotropic model, its ease of implementation in finite element schemes, and its excellent ability to model the mechanical heterogeneity of tibial AC under a physiologically-relevant strain rate suggest that it could serve as a viable constitutive relation for healthy human knee AC. It should be noted that the data used for this study were taken from middle-aged, non-overweight Caucasian females whose activity level prior to death and natural limb alignment were unknown. Therefore, further extrapolation of this model may currently be limited. Future work will investigate the ability of the model to predict other deformation states in more diverse populations.

ACKNOWLEDGEMENTS

This project was funded in part by a Rackham Graduate School Student Research Grant. Ms. Deneweth is funded by the Department of Defense through the NDSEG Program.

REFERENCES

1. Lai W, et al. *J Biomech Eng*. **115**:474-80, 1993.
2. Lu XL, et al. *J Biomech* **43**:673-9, 2010.
3. Mizrahi J, et al. *Biorheology.* **23**:311-30, 1986.
4. Brown CP, et al. *Proc. Inst. Mech. Eng. H J Eng. Med.* **223**:643-52, 2009.
5. Deneweth JM, et al. *J Orthop Res*, 2012.
6. Arruda EM, et al. *J Mech Phys Solids*. **41**:389-412, 1993.
7. Palmer JS, et al. *Acta Biomater.* **4**:597-612, 2008.
8. Bischoff JE, et al. *J Appl Mech-T ASME*. **69**:570-9, 2002.