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R. McN. ALEXANDER's TARGET POINT FOR GROUND REACTION FORCES

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SUMMARY
In 1978, on the basis of force plate records, Jayes and Alexander found out, that irrespective of gait and speed, the force exerted by mammalian foot on the ground changes direction during the contact phase of each particular leg so as to remain in line with a point fixed relative to the animal's trunk and located well above the limb apex. I call it "Alexander's point". The phenomenon was never explained theoretically.
Here I suggest a model of the steady horizontal locomotion, which shows that in order to minimize the sum of mechanical work magnitudes in all the limb joints at any instance of the contact phase, the ground reaction vector should follow the limb articulations in the order they attain the smallest angular deflection from the vertical. In the case of the hindlimb it should follow the knee, hip, and finally the ankle. This produces the saw-shaped graph of the tangent of the angle of the ground force deflection from the vertical with two abrupt changes of the sign of its horizontal component – when its vector jumps from the knee to the hip and from the hip to the ankle.
It is suggested that the limb muscles are unable to contract fast enough to perform such abrupt shifts of the ground force direction, and the simplified pattern is actually used. It is modeled as the least square linear regression trend of the saw-shaped graph, which appears to be very close to experimental evidence for Alexander's point.

INTRODUCTION

The major problem of the chain-like animal limbs in terrestrial locomotion is that they cannot work like a wheel, because the ground reaction force can be aligned with only one of its articulations at a time, while the other ones must be balanced by muscular activity. This activity, in its turn, is associated with mechanical work, because the joint angles inevitably change, and active muscles shorten or lengthen producing excessive (as compared to the wheel) positive or negative work, respectively.

The experimental studies of ground reaction forces in quadrupeds and bipeds are numerous, but generalizations of these data are few. The most important one was drawn from experiments on dog and sheep by A. S. Jayes together with R. McN. Alexander, the famous British biomechanician. They found out, that irrespective of gait and speed, the force exerted by a foot on the ground changes direction during the contact phase of each particular leg so as to remain in line with a point fixed relative to the animal's trunk and located well above the limb apex (i.e. acetabulum or the shoulder girdle) [1]. I shall call it "Alexander's point". This phenomenon remains unaccounted and is almost forgotten, despite of its probable significance for minimization of energy expenditures.

METHODS

The research is based on the following theoretical assumptions. For any instance of the contact phase, a straight line (*a*) through the foot can be built, along which the ground reaction force would act with the minimum total mechanical work in the limb joints. Suppose the weightless limb and rigid trunk, which undergoes no pitching, rolling or yawing. In this case, the line (*a*) is perpendicular to the instantaneous velocity of the center of gravity of the trunk, and if the ground reaction is applied to the foot along this line, the total mechanical work in the limb joints is zero. Consider the simplest case of steady walking, when the velocity of the center of gravity is horizontal. Then, the line (*a*) is vertical. Although the total mechanical work is zero, the work in some joints is positive, while in the other ones it is negative – muscles of these joints work against each other [2,3].

Further we shall consider the flat (parasagittal) legs typical to dogs, sheep and other therians as well as birds, because it is unknown if Alexander's point exists in the sprawling quadrupeds such as reptiles. Imagine now the straight lines passing through the foot and each one of the limb articulations (Figure 1). Generally, the instantaneous angles between these lines and the line (*a*) in the parasagittal limb plane are different. For example, when mammalian hindlimb hits the ground, its knee deflects from the vertical by smaller angle than the hip and the ankle.

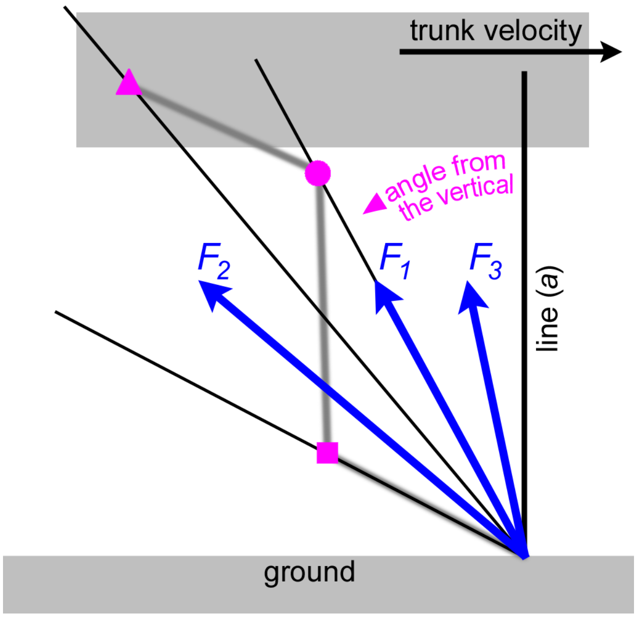

Figure 1: Theoretical model explained in the text. Ground reaction forces F_1, F_2, F_3 have equal vertical components.

In steady walking, the ground reaction vertical component acting along the line (*a*) is given at any instant of the contact phase by the balance with trunk gravity. If we regard the sum of instantaneous mechanical work magnitudes in all the joints as the energy optimum, it can be shown that the best solution is to align the ground reaction with the articulation, which deflects from the line (*a*) by the least angular magnitude (F_1 on Figure 1). When the ground reaction deflects more than this (F_2 on Figure 1), it causes greater decrease or increase in the horizontal kinetic energy of the trunk, and when it deflects less (F_3 on Figure 1), the extra energy expenditure occurs due to the work of muscles against each other, which reaches its maximum when the ground reaction acts along the line (*a*).

RESULTS AND DISCUSSION

Jayes and Alexander [1] have illustrated the true change of ground force angular deflection from the vertical in the hindlimb contact phase of a walking dog by plotting the tangent of this angle against time. Their Fig. 11b is reproduced here by black diamonds on Figure 2. The negative tangent corresponds to the braking horizontal component of the ground reaction, typical to the first half of the contact phase, while the positive tangent implies horizontal acceleration in the second half. The fact that the graph is almost rectilinear and its gentle inclination, were those reasons which lead to the conclusion of the existence of Alexander's point above the hip [1].

I have added to this graph the theoretically optimum tangent values (pink points on Figure 2). For this purpose, I have analyzed a slow-motion video sequence of a dog walking over the treadmill, and deduced for every frame of the hindlimb contact phase the energy-saving direction of the ground reaction as it is predicted by the above theoretical consideration: the ground reaction vector should follow the hindlimb articulations in the order they attain the smallest angular deflections from the vertical. At first it follows the knee (circles on Figure 2), then the hip (triangles on Figure 2), and finally the ankle (squares on Figure 2). So, the theoretical graph as a whole looks like a trident saw. The segments of the saw are steeper than the experimental graph because all the three limb articulations are well below Alexander's point.

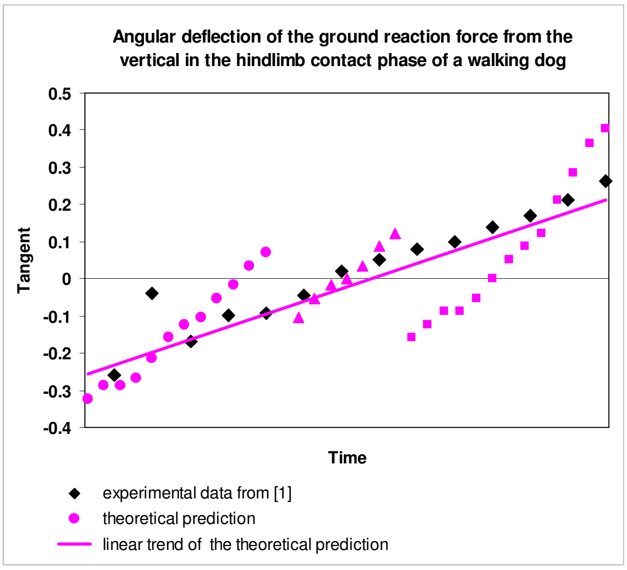

Figure 2: Comparison of ground reaction deflections from the vertical in theory and experiment.

Then I have calculated the least square linear regression trend of the full set of theoretical points and depicted it on the same graph (pink line on Figure 2).

Except for initial impact disturbance in the experimental graph (view its second black diamond), the regression line for the theoretical data set appears to be strikingly close to the true values. This regression line can be regarded as the theoretical explanation for the existence of Alexander's point, but why is it used by dog, sheep and, most probably, other therians instead of the more energy-saving saw-shaped pattern?

The reason may be in the insufficient rapidity of limb muscles to perform abrupt changes of the ground reaction force. Similar explanation was suggested by Jayes and Alexander, when they found out that chelonian walking gait is not as stable as it could be with the more abrupt changes of the ground reaction [4]. Obviously, mammalian muscles are faster than chelonian ones, but the saw-shaped pattern of the ground reaction, which could be desired by mammals, is much more abrupt than what was supposed as the best solution for chelonians. In chelonians only the magnitude of the vertical component of the ground reaction was to be changed stepwise if their muscles were faster, not the direction of its horizontal component, which in the saw-shaped model has to be instantly switched from positive to negative twice during the contact phase. Even mammalian muscles cannot switch on and off immediately, which ability is necessary to change the target point of the ground reaction from the knee to the hip and finally to the ankle joint. The penalty for this muscular sluggishness is the additional work of muscles against each other, inevitably associated with keeping Alexander's point, which however can be reduced by peculiar limb kinematics and with the help of two-joint muscles [3].

CONCLUSIONS

Alexander's point exists for the two reasons: (1) the tendency to minimize at any instance of the contact phase the sum of mechanical work magnitudes in all the limb joints, and (2) the inability of mammalian musculature to contract fast enough to fulfill this condition.

ACKNOWLEDGEMENTS

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