MODELLING OF BRAIN TISSUE: FRACTIONAL VISCOELASTIC MODEL AND EXPERIMENTS

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INTRODUCTION

During the last decades, an increasing effort has been put on both brain tissue modelling and fractional calculus. This mathematical field provides a very suitable tool for the modelling of viscoelastic materials thanks to its non-local property. A global biomechanical model of the brain using fractional-based viscoelasticity could find applications in neurosurgery and haptic devices design as well as in car manufacturing to evaluate the possible trauma due to an impact. To this purpose, it should reproduce the behaviour of the brain for several strain-rates.

METHODS

Assuming the incompressibility of the brain, classical hyperviscoelastic models can be written in the following form:

$$\mathbf{S} = 2\mathbf{S}_w - p\mathbf{C}^{-1} \tag{1}$$

$$\mathbf{S}_w = \frac{\partial}{\partial \mathbf{C}} \int_0^t J_{rel}(t-\tau)\frac{d\Phi}{d\tau}\, d\tau \tag{2}$$

where $\mathbf{S}$ designates the second Piola-Kirchhoff stress tensor, $\mathbf{C}$ is the right Cauchy Green tensor, J_{rel} stands for a relaxation function, Φ is a strain-energy density function and p an unknown hydrostatic pressure to be determined by the boundary conditions and the equilibrium equations. If the relaxation function is written as a sum of decreasing exponentials, expression (2) can be written as the linear differential equation (3)

$$D^{(m)}\mathbf{S}_w + \sum_{k=0}^{m-1} c_k D^{(k)}\mathbf{S}_w = \mathbf{G}(\mathbf{C}, \dot{\mathbf{C}}, \ddot{\mathbf{C}}) \tag{3}$$

$\mathbf{G}$ is related to the instantaneous loading.

Although its origin goes back to the end of the 17th century, it was not before 1920 that the fractional calculus was used in a physical framework. For years, it has begun to spread in the fields of soft tissues and polymers modelling. Amongst the existing definitions of the fractional derivative, we chose to use Caputo's one [1].

The idea is then to replace the ordinary differential equation by a real-order one. The complete development for $m = 2$ can be found in [2]. The fractional differential equation in this particular case writes:

$$D^{(\alpha)}\mathbf{S}_w + b\mathbf{S}_w = \mathbf{H}(\mathbf{C}, \dot{\mathbf{C}}, \ddot{\mathbf{C}}) \tag{4}$$

where $D^{(\alpha)}$ stands for the derivative of real order α and $\mathbf{H}$ is a linear function of its arguments. The model described by equation (4) counts 6 parameters. The identification was performed on simple compression tests data at different strain-rates available in the literature [3].

RESULTS AND DISCUSSION

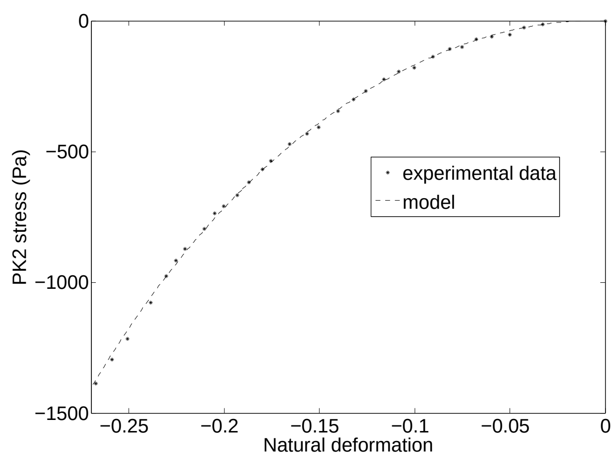

Figure 1: PK2 stress vs natural strain for the unconfined compression experiment (strain rate = $0.64\ 10^{-2}s^{-1}$). The experimental data comes from [3]

Figure (1) shows that the model almost perfectly fits the experimental data. With its 6 parameters, the model has proved to be accurate with the strain-rate varying over two orders of magnitude. However, numerical simulations have shown that other kinds of experiments are needed in order to characterize the material properties. Compression, relaxation and cyclic tests are currently carried out at the University of Liege. We use a 3d video reconstruction system to estimate the deformed geometry of the samples.

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REFERENCES

1. Caputo M, *The Geophys J Astro Society*. **13**:529-539, 1967
2. Libertiaux V & Pascon F, to be published in the *J Comp Appl Math*
3. Miller K & Chinzei K, *J. Biomech*.**30**:1115-1121, 1997
4. Podlubny I, *Fractional differential equations*, London Academic Press, 1999