# A SIMULATION OF TIME DELAY BETWEEN ACCELERATION AND ADDED FORCE IN SWIMMING 

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## INTRODUCTION

Swimmers propell under the action of a cyclic force. It is important to know the relation of peaks between added forces and accleration of body in swimming. We simulate a motion by solving a simplified nonlinear eqaution. In the cyclic fluctuations, acceleration of the center of gravity of body becomes an index for determining the useful timing for an addied force. This study describes the time difference between accerleration and working force.

## METHODS

Supposing a cyclic force $F(t)$ is acting on a body of the mass $m$ under the action of drag proportional to the squared velocity, the governing equation is expressed approximately,
$\left(m+m^{\prime}\right) \frac{d^{2} x}{d t^{2}}=F(t)-\frac{k}{2}\left|\frac{d x}{d t}\right| \frac{d x}{d t}+$ BassetTerm ,
where $m^{\prime}$ is the added mass and the Basset Term means the memory effect in unsteady flow. If the body of mass $m$ travels at a constant speed $V_{0}$ under the action of constant force $F_{0}$, the relation $F_{0}=k V_{0}{ }^{2} / 2$ holds. The second term on the right-hand side of Eq.(1) is the drag force proportional to the squared velocity. Omitting the memory effect, we have the non-dimensional form corresponding to the dimensional one (1),
$\frac{d^{2} x}{d t^{2}}=F(t)-\left|\frac{d x}{d t}\right| \frac{d x}{d t}$,
where the characteristic time $T_{0}$ was chosen as the time when the velocity damps to a half of the initial velocity in the absence of the added force. Moreover for the characteristic length $L_{0}$ we have chosen such that $L_{0} / V_{0} T_{0}=1$.

Setting $x=0$ and $d x / d t=1$ as initial values, we solved Eq.(2) by use of Runge-Kutta method. The cyclic force $F(t)$ was chosen as

$$
F(t)=1+a \sin \omega t
$$

The amplitude is determined so as to generate not also thrust but also weak drag, or other words it becomes greater than unity a little.

## RESULTS AND DISCUSSION

We first show the results when $a=1.2$ and $\omega=4$. In Figure 1 time variations of force, velocity and acceleration were plotted. The second peak of acceleration was denoted by $t=t_{1}$ and that of force by $t=t_{2}$. It should be noted that the peak of acceleration appears prior to that of force peak. The phase of velocity variation delays largely compared with those of acceleration and force. The time difference $t_{1}-t_{2}$ is small for $\omega=4$. However, it
becomes larger as the frequency $\omega$ ceases to zero as shown in Figure 2.


Figure 1: Velocity, acceleration and force variation as functions of time for $\omega=4$. Times $t=t_{1}$ and $t=t_{2}$ are the second peaks of the acceleration and the force, respectively.


Figure 2: Response of propulsive force to acceleration.
According to the MAD system[1], $F_{0}$ is given as 93.2 N for $V_{0}=1.9 \mathrm{~m} / \mathrm{s}$. Supposing $m=75 \mathrm{~kg}$ and $\mathrm{m}^{\prime} / m=0.2$, we have $T_{0}=1.83 \mathrm{~s}$. For this case, the time difference is calculated as 0.18 s . The difference corresponding to the case of $\omega=6$ is 0.092 s .

## CONCLUSIONS

It was found that the phase of acceleration proceeds compared with that of added force. The difference becomes bigger as the frequency decreases. The phase difference is important in knowing the moment that the force works effectively.

## REFERENCES

1. Toussaint HM, et al., J Biomech.. 37:1655-1663, 2004.
