BIOLOGICALLY RELEVANT MODEL OF ELASTIC RESPONSE OF TENDON IN AXIAL TENSION

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INTRODUCTION

Tendons subjected to axial tension present a stress-strain curve characterised by a non-linear toe region followed by a linear region, most likely arising from uncrimping and/or recruitment and subsequent stretching of the collagen fibres. While numerous models for such behaviour exist, each is inadequate in some regard. The constants needed for Viidik's mechanical analogy [1], for instance, lack direct biological counterparts. Comninou and Yannas' model [2] is limited to sinusoidal crimp shapes and is dependent on crimp amplitude and period. Kastelic's model [3] relies on an unsupported crimp angle distribution function, while Fung's exponential function [4] applies only within a limited strain range. The aim of this study is to determine whether a new model may fit experimental stress-strain data for tendon through use of just three biologically relevant parameters.

METHODS

The model was developed based on work by Buckley, Lloyd, and Konopasek [5], referred to here as BLK. BLK conducted numerous numerical simulations in which a slender filament with planar crimp was modelled as an extensible elastica. The load-extension relation was determined as a function of three dimensionless parameters: R, D, and q. Slenderness ratio R is the radius of the cross-section of the filament divided by the contour length for one wavelength, crimp level D is the ratio of the filament length to the contour length, and shape parameter q describes the shape of the filament and ranges from $q = -\infty$ for arcs of circles to $q = \infty$ for a zig-zag crimp pattern. Intermediately, $q = 0$ describes a cubic variation of curvature, or a near-sinusoidal crimp shape. Filament load-extension curves were expressed as $\Phi(\xi)$, where Φ is the ratio of force to the bending rigidity divided by the square of the contour length, and ξ is a measure of the proximity (along the strain-axis) of a given point on a $\Phi(\varepsilon)$ load-extension curve to the linear portion of the curve. BLK made the key observation that for all $D, R << 1$, and $q \neq \infty$, $\Phi(\xi)$ was only weakly dependent on shape and crimp level, providing a universal curve.

The present work included application of this approach to experimental stress-strain data for axial tensile loading in the fibre-aligned direction of nine bovine digital extensor tendons, preconditioned in 10% sucrose solution and loaded to failure at a nominal strain rate of 0.6%/s. To fit the data, a set of global parameters was found to describe the universal $\Phi(\xi)$ curve from the BLK numerical results for all D, $R \leq 0.05$, and $q \leq 0$. Then for each set of stress-strain data, the slope m and projected strain-axis intercept ε_0 of the linear region were found and fitting was done to determine the slenderness R required to map the data onto the universal curve. Thus, each stress-strain curve was fit based on three physically relevant parameters: m, ε_0, and R.

RESULTS AND DISCUSSION

The model was able to fit diversely shaped stress-strain curves, capturing various degrees of curvature of the toe region and differing slopes of the linear region (Figures 1 and 2). R found in fitting ranged from 0.0049 to 0.0195, compared to the estimated minimum R of 0.0221 based on microscopically measured values of 60° for initial crimp angle [6], 240 μm for initial crimp period [7], and 21.2 μm for fibre diameter [8]. Though results were promising, this study was limited to nine bovine tendons, necessitating future validation of the model using additional data sets for both bovine and human tendons.

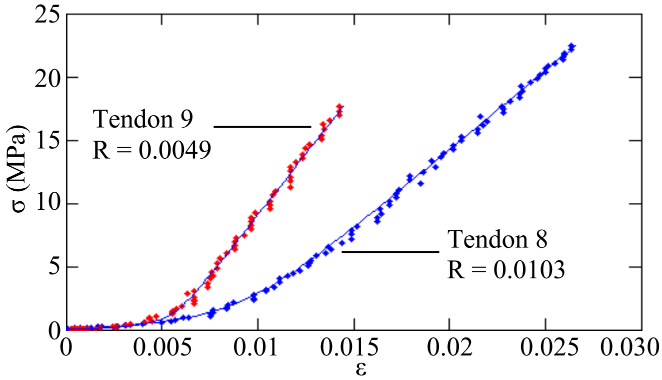

Figure 1: Fit of model (lines) to experimental data (dots) for two bovine tendons with well-defined toe regions.

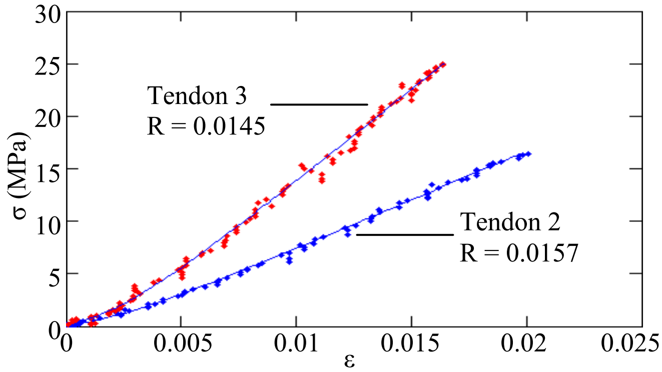

Figure 2: Fit of model (lines) to experimental data (dots) for two bovine tendons without highly defined toe regions.

CONCLUSIONS

This model improves upon previous models for the elastic response of tendon in axial tension, capturing both the toe and linear regions of experimentally obtained stress-strain curves through use of only the slope and strain-intercept of the linear region of the curve and the slenderness ratio of the collagen fibres dominating the mechanical response.

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