# A MATHEMATICAL DEFINITION OF FEASIBLE FINGER POSTURES AND MOVEMENTS

[1] Saurabh Mahapatra, [2]John Guckenheimer and [1]Francisco J. Valero-Cuevas
[1] Neuromuscular Biomechanics Laboratory, [2] Department of Mathematics, Cornell University, NY, USA,
email: {SM387, JMG16, FV24}@cornell.edu

## INTRODUCTION

It is not well understood why or how neurological diseases or injuries so easily disrupt the delicate interactions among finger muscles that produce finger postures and slow movements. Thus, we lack clear therapeutic approaches to, say, shape the hand for grasp following stroke. A first-order dynamical systems approach is applicable to this neuromuscular control problem because finger postures are combinations of joint angles, and slow finger movements are damped during transitions between postures. Our long-term goal is to explicitly define the manifold of feasible finger postures in state-space as a function of the continuous muscle inputs that drive the fingers to various configurations during slow movements. As a first step, we develop here, the mathematical tools to find such manifolds and their stable points. While our approach is reminiscent of the *deformable membrane* analogy of the equilibrium-point hypothesis, our purpose is quite different: to determine whether first-order dynamical equations derived from basic biomechanical and muscle mechanics concepts can begin to explain experimental measurements of finger postures and slow movements.

## METHODS

To facilitate visualization of our methods, we present the example of a tendon-driven, 2-joint, planar finger with anatomically inspired tendon routing [1]. For quasi-static motions, muscles are modeled as simple force-length actuators with the resting lengths as parametric inputs and the tendons are modeled as elastic bands. Using geometrical constraints obtained from tendon routing, we obtain relationships between tendon stretch/excursions and joint angles (generalized coordinates). This relationship we obtain is called the transmission matrix ***G***. For a given set of muscle resting length inputs, the potential energy of the system can be written as a non-linear function of tendon stretch/excursions and joint angles. This potential energy function is a high dimensional manifold in the generalized coordinate space whose maxima/minima give the equilibrium postures for the finger. These maxima/minima when plotted as a function of the muscle input parameters also give a manifold, the local and global properties of which determine if the system is controllable locally and/or globally. We use results from manifold theory to study such properties. For example, the Implicit Function Theorem lays down the sufficient conditions for a functional relationship to exist between the muscle inputs and the output generalized coordinates [2].

## RESULTS AND DISCUSSION

For the above example, we found that the transmission matrix ***G*** determines the controllability of the system. A necessary and sufficient condition for such a system to be controllable is that the rank of the matrix ***G*** should be equal to the number of joints (generalized coordinates). For two test cases, the results are shown in Figure 2. We found that the cross-over tendon m3 shown in Figure 1 (analogous to the extensor mechanism of the fingers) plays a major role in determining this rank for ***G***, hence showing the importance of anatomically faithful models.

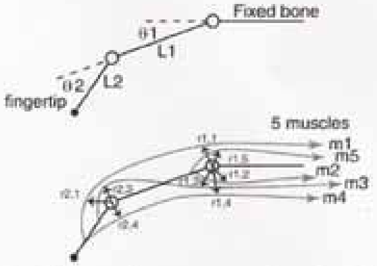

**Figure 1:** The tendon geometry for the 2-link 5-muscle finger.

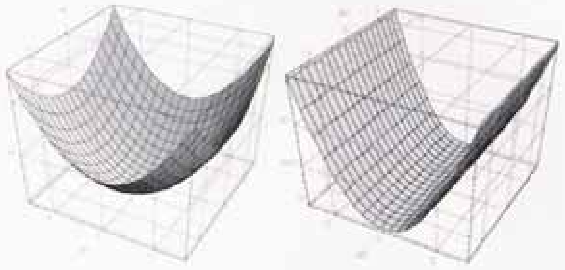

**Figure 2:** Left: The potential energy function manifold is a paraboloid in the joint angle space when rank (***G***) =2. The system is fully controllable along the two degrees of freedom. Right: The potential energy function manifold reduces to a parabolic cylinder when rank (***G***) =1. There is loss of controllability along the axis of this cylinder.

Having developed the ability to calculate the manifolds that predict stable finger postures and gradients for finger movement, our next step is to create anatomically faithful 3D models of the human finger with complex tendon geometry.

## REFERENCES

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