MODELING CEREBRAL ANEURYSM FORMATION AND ASSOCIATED STRUCTURAL CHANGES

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INTRODUCTION

Intracranial aneurysms (ICA) are saccular dilations of cerebral arteries, most commonly found at the apices of arterial bifurcations on or near the Circle of Willis. Clinical and histological evidence support the hypothesis that an ICA develops from a local weakening of the wall, possibly due to mechanical or biochemical processes. It is hypothesized, that as a result, the bifurcation apex (A) bulges into an early stage aneurysm 'bleb' with no identifiable neck (B), and then into an aneurysm with a clear neck region (C). It is further hypothesized that the process from (B) to (C) is accompanied by collagen degradation and deposition. When the two processes are well balanced, the ICA is secured from rupture and develops into a complex aneurysm, exhibiting biological responses such as calcification and thrombosis (D) [4].

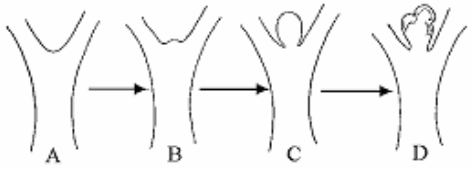

Figure 1 Stages of Aneurysm Development

We recently developed a dual-mechanism constitutive equation capable of modeling collagen recruitment and disruption of the IEL [2,3] based on the nonlinear inelastic behavior of cerebral arteries reported in [5]. This is the first ICA wall model that takes into account the IEL disruption, commonly observed in histological studies of ICA walls. This model was able to predict the "sac like" shape of the ICA and other important features of early aneurysm formation [1,2]. In this work, we extend the model to include the collagen degradation and synthesis.

CONSTITUTIVE MODEL

Three mechanisms are employed in this model representing the passive mechanical elements of artery and aneurysm walls: elastin, recruited collagen and newly synthesized collagen. Each mechanism is activated at a different deformation level and thus has a unique reference configuration. A scalar function s that depends on the deformation gradient $\mathbf{F_1}$ relative to the stress free initial reference configuration κ_1 is introduced to quantify the level of deformation. The activation and deactivation of the mechanisms commence when s reaches critical values s_a, s_b, s_c and s_d corresponding to arterial collagen activation, elastin breakage, collagen degradation and the synthesis of new collagen, respectively. Following [2-3], we suppose that $s_a < s_b$.

At sufficiently low loads, $s \le s_a$, only the elastin mechanism, which is presumed to depend on $\mathbf{F_1}$, is load bearing. When $s = s_a$, the collagen mechanism is activated in the mechanical response and the response is a function of the deformation gradient $\mathbf{F_2}$ relative to configuration κ_2 occupied by the body at s=sa. The elastin disrupture is modeled by irreversible termination of the elastin response at $s=s_b$. Collagen degradation will be specified to commence when $s=s_c$. The gradual degradation is carried out by assigning a monotonically decreasing function b(s), which is chosen such that $b(s_c)=1$, that represents the volume fraction of the remaining material (e.g. [6]). Furthermore, collagen synthesis is modeled as continuous integration of a new mechanism that begins when $s=s_d$. Assuming the existence of an incompressible isotropic exponential type strain energy function W_3, the extra stress tensor due to collagen synthesis is

$$\tau_3 = 2\int_{s_d}^{s} a(\hat{s})\frac{dW_3}{d(I|_{\hat{s}})}\underline{B}|_{\hat{s}}\, d\hat{s} \qquad (1)$$

(see, e.g.[6] for more details) where a(s) is related to the rate of synthesis. Following [1-3], the other two mechanisms are assumed to be described by strain energy density functions W_1 and W_2, similar to W_3. For purely monotonically increasing s, the Cauchy extra stress tensor is given by,

$$\begin{aligned}
&\boldsymbol{\tau} = \boldsymbol{\tau}_1 && \text{for s in } [0,s_a)\\
&\boldsymbol{\tau} = \boldsymbol{\tau}_1 + \boldsymbol{\tau}_2 && \text{for s in } [s_a,s_b]\\
&\boldsymbol{\tau} = \boldsymbol{\tau}_2 && \text{for s in } [s_b,s_c] \qquad (2)\\
&\boldsymbol{\tau} = b(s)\,\boldsymbol{\tau}_2 && \text{for s in } [s_c,s_d]\\
&\boldsymbol{\tau} = b(s)\,\boldsymbol{\tau}_2 + \boldsymbol{\tau}_3 && \text{for } s > s_d
\end{aligned}$$

where $s = I_1-3$, $\boldsymbol{\tau}_1 = 2\, dW_1/dI_1\, \mathbf{B}_1$, $\boldsymbol{\tau}_2 = 2\, dW_2/dI_2$ and $\boldsymbol{\tau}_3$ is given by (1). As is shown in [1-3], the strain energy functions W_1 and W_2 are given by $W_1 = C_1(e^{\gamma_1(I_1-3)}-1)$ and $W_2 = C_2(e^{\gamma_2(I_2-3)}-1)$ where I_1, I_2 are the first invariants of the left Cauchy Green tensors, $\mathbf{B}_1=\mathbf{F}_1\mathbf{F}_1^T$ and $\mathbf{B}_2=\mathbf{F}_2\mathbf{F}_2^T$, respectively. The material constants C_1, C_2, γ_1, and γ_2 were obtained using the experimental data of Scott *et al* [5].

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ACKNOWLEDGEMENTS

The author RW acknowledges the support from the American Heart Association (AHA 0325737U)