

## APPROXIMATION OF BALANCED LANDINGS IN GYMNASTIC DISMOUNTS

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### INTRODUCTION

Gymnastic dismount landing phases have two parts: a short duration impact phase when foot-mat contact forces are high, and a subsequent balancing phase during which the gymnast exerts active control over the resulting post-impact velocities to achieve a desired motionless terminal configuration. Simple impulse-momentum approximate models for the two phases can help in understanding what pre-impact motion states are controllable and thus are feasible targets for the end of flight.

### METHODS

Approximate quasi-rigid-body gymnast models are used for both impact and balance phases. Impact is characterized by larger forces over a shorter duration ( $\Delta t = 0.04$  sec) than the balancing phase [1]. Although large impulses at the feet create similar joint impulses and cause the body to deviate slightly from its presumed rigid pre-impact configuration, the short impact period motivates the approximation that the body remains rigid. Nevertheless, impact is long enough to make total rigid-body rotation ( $\sim 0.3$  rad) non-negligible. Whole-body rotation is included by assuming that angular velocity decreases linearly during impact and that the rigid body rotates about the ankle. Non-instantaneous linear and angular impulse and momentum relations approximate the impact phase. Horizontal and vertical impulses,  $\mathcal{G}_x$  and  $\mathcal{G}_y$ , applied to the gymnast's feet cause them to stop [1] and decrease body angular velocity according to

$$\mathcal{G}_y = m(v_{yf} - v_{yo}) + mg\Delta t_i \quad (1)$$

$$\mathcal{G}_x = m(v_{xf} - v_{xo}) \quad (2)$$

$$I_{cm}(\omega_f - \omega_o) = r(\mathcal{G}_x \cos \theta_{avg} + \mathcal{G}_y \sin \theta_{avg}) \quad (3)$$

where  $m$  is gymnast mass,  $r$  the distance from feet to center of mass (cm),  $g$  gravity,  $\theta_{avg} = (\omega_o + \omega_f)\Delta t / 4 + \theta_o$  the average angle between the line from ankle to cm and vertical during impact,  $I_{cm}$  the whole body inertia about the cm,  $\omega$  the angular velocity,  $v_x$  and  $v_y$  the horizontal and vertical velocities, and subscripts  $o$  and  $f$  denote prior to and after impact.

In a balanced landing, impulses reduce the angular velocity to an amount that can be controlled by multiple muscle torques during the balance phase. While the impulses may cause post-impact angular velocity to be exactly zero, balancing is possible over a range of landing configurations and velocities. Moreover, post-impact balance control is generally exerted at all joints and is a multi-input multi-output control problem. A conservative approximate model of balance allows the gymnast to attenuate the post impact angular velocity while remaining rigid and using only the ankle moment created by pressing the toe or heel into the ground. The ankle moment is limited since the ground reaction force ( $\sim mg$ ) must pass

through the foot. Minimum and maximum possible post-impact angular velocities  $\omega_{min}$  and  $\omega_{max}$  are calculated by solving differential equations for this motion,

$$I_{cm}\ddot{\theta}_{min} = mgr \sin \theta_{min} + mgd_t \quad (4)$$

$$I_{cm}\ddot{\theta}_{max} = mgr \sin \theta_{max} - mgd_h \quad (5)$$

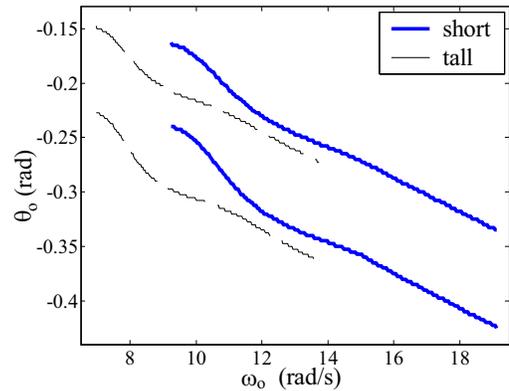
where  $d_t$  and  $d_h$  are horizontal distances from ankle to toe and heel, respectively.

### RESULTS AND DISCUSSION

Short and tall gymnasts with properties and bar release conditions shown in Table 1 can complete 3 and 2 dismount somersaults, respectively, and are configured differently at impact to reduce angular velocity enough to balance [2].

**Table 1:** Gymnast properties and bar release conditions

Ht (m)	Mass (kg)	$I_{cm}$ (kg·m <sup>2</sup> )	$\omega$ (rad/s)	$H_{cm}$ (kg·m <sup>2</sup> /s)
1.57	62.38	9.20	9.84	90.57
1.71	69.30	12.57	8.96	112.71



**Figure 1:** Ranges for pre-impact contact angles for balanced landings differ for short and tall gymnasts

The ranges of all possible landings for the short and tall gymnast's given their angular momentum  $H$  at bar release are shown in Figure 1. If the athlete has a larger impact angular velocity  $\omega_o$  and small inertia, she must contact the ground with a larger body angle relative to vertical  $\theta_o$  than an athlete with larger inertia and smaller angular momentum because she will rotate further during impact. The short gymnast is able to balance over a larger range of  $\omega_o$  than the tall one. With a given  $\omega_o$ , the tall gymnast is able to balance with a slightly larger range of  $\theta_o$  due to a relatively larger foot size which increases ankle moment (Figure 1). Although the range of possible  $\theta_o$  decreases for small values of  $\omega_o$ , the athlete is in controllable landing configurations longer since she rotates more slowly, making a balanced landing more likely.

### REFERENCES

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