ESTIMATING THE AXIS OF A SCREW MOTION FROM NOISY DATA — NEW METHOD BASED ON PLÜCKER LINES

[1] Koon Kiat Teu, [2]Wangdo Kim
[1]School of Mechanical & Production Engineering, Nanyang Technological University, Singapore
[2]Biomechanics Laboratory, Legacy Clinical Research & Technology Center; email: wkim@biomechresearch.org

INTRODUCTION

Screw axis has been employed to describe joint kinematics mainly in clinical assessments. It is intuitive that the geometry of the screw axis of a spatial displacement can be fully and easily studied using Plücker's coordinates that define the line directly [1]. One of the fundamental problems in calculating screw axis is noisy data. The new algorithm presented in this paper takes advantage of set of line patterns, which are derived from point data using dual number relationship. The method involves mapping a set of lines to the corresponding lines after movement. The mapping of vectors is made possible by the use of dual transformation matrix. The dual transformation matrix (DTM) has been shown to be effective means of three-dimensional *line* transformation in displacement analysis [2].

METHODS

The method in this paper involves obtaining the DTM from coordinated points. This method has been mentioned in [2]. After solving for DTM $[\hat{R}]$, we solve for the dual vector associated with DTM using the eigenvector.

$$[I - \hat{R}]\hat{V} = 0, \quad (1)$$

and seek solution other than $\hat{V} = 0$. This is easily done if we separate it into the pair of vector equations compromising the primary and dual component respectively

$$[I - R]\mathbf{V} = 0$$

$$[I - R]\mathbf{W} = [D]\mathbf{V} \quad (2)$$

Where $[D]$ is the skew-symmetric matrix defined by $[D]\mathbf{V} = \mathbf{d} \times \mathbf{V}$ for any translation and we used the property of $[R]\mathbf{V} = \mathbf{V}$.

The proposed method was tested on simulated data. The simulations follow closely to that of [4].

RESULTS AND DISCUSSION

The simulation results in [4] ("new" and "FHA" method) were used as references for the proposed algorithm. The results of the "new" methods were superimposed to our simulation results in Figure 1 and 2 as a comparison. The general trend of the error follows that of those reference methods. The errors in direction and position were lower when there's significant noise. For skin movement artefact simulation, the estimation of the position using the proposed method was better. The error in direction is reasonably good and follows closely to that of the "FHA" method. The simulation results showed that the proposed method worked well, especially in estimating the direction of the screw axis when noise is present. The mapping of vectors gives a better estimate of the rotation than mapping of points data. The results are highly significant as they demonstrate an advantage of using line instead of points.

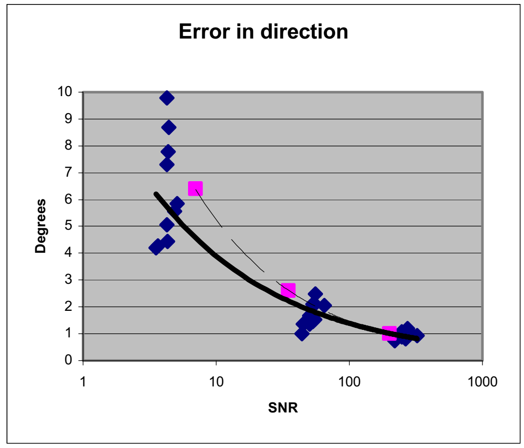

Figure 1: Mean error in the estimated direction of the axis plotted against the SNR (Signal To Noise Ratio). The solid lines correspond to the presented method and the dotted lines to the "new" method.

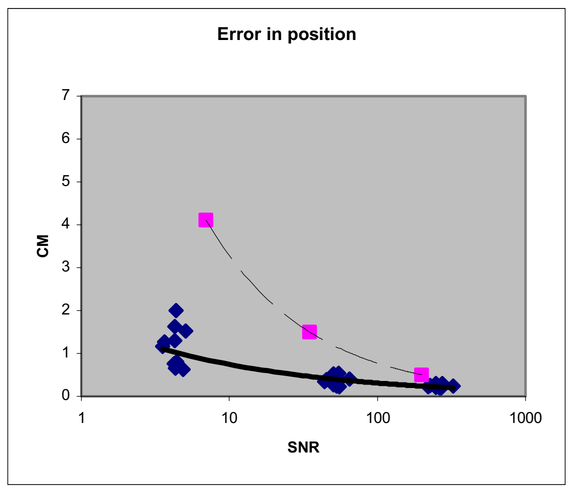

Figure 2: Mean error in the estimated position of the axis plotted against the SNR

CONCLUSIONS

In our study, the estimation of screw axes from a set of Plücker lines clearly outperforms the conventional methods using points data. This opens the range of new practical applications of Plücker lines in bio-kinematics situation. The method provides an alternative analysis tool for estimation of screw axes and can be applied to any actions such as gait analysis in clinical setting to estimate the screw axis.

REFERENCES

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