SCALING AND JUMPING: GRAVITY LOSES GRIP ON SMALL JUMPERS

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INTRODUCTION

The current view on the scale effects on jumping is that (1) jumping performance is independent of size in the absence of air friction: all animals would achieve the same jump height (rise of the body center of mass while airborne) if they were geometrically similar and delivered the same amount of work per kg body mass during the push-off (Borelli's Law), and (2) in the presence of air friction smaller jumpers are at a disadvantage, because they waste relatively more energy on air friction than larger animals due to their larger surface to volume ratio.

It can not be explained from an evolutionary view why jumping locomotion is primarily adopted by small animals, such as insects. Besides, jumping is a battle against gravity and for static situations, i.e. standing with flexed legs, defying gravity becomes easier with decreasing size because the moment of gravity that needs to be counteracted decreases at a higher rate than the muscle moment (L^4 and L^3 respectively, where L is the scaling factor for length). It is evaluated analytically and numerically how this mechanical advantage translates to dynamic situations and affects jumping performance as a function of size.

METHODS

On the basis of energy balances for geometrically similar jumpers consisting of a point mass and massless legs, it is shown analytically that Borelli's Law is wrong. With the same mass specific work (work per kg body mass), smaller jumpers achieve higher take-off velocities and hence greater jump heights.

$$v_{take-off} = \sqrt{2(W_m - Lg\Delta h)}$$

where $v_{take-off}$ is take-off velocity, W_m is mass specific work, g is the acceleration due to gravity, Δh is the height gained prior to take-off and L is the scaling factor for length.

To assess how the relationship between size and jumping performance contributes to our understanding of real jumping animals, numerical simulations were conducted using a more realistic generic jumper model [1]. One hundred geometrically similar bipedal jumpers ranging from $7{*}10^{-6}$ kg to 70 kg were modeled. Jumpers were actuated by constant knee extensor torques that scaled with mass, so that all jumpers produced the same amount of mass specific work over the same angular knee-extension.

RESULTS AND DISCUSSION

Smaller jumpers achieved greater jump heights than larger jumpers. Absolute jump height increased by 70% when scaling down a jumper from 70 kg to 0.7 g. Figure 1 shows the amount of mass specific work delivered by each jumper during push-off, as well as how it was expended. A division is made in (1) effective kinetic energy (energy due to vertical velocity of the body centre of mass), (2) potential energy and (3) 'useless' kinetic energy, energy that does not contribute to jump height (e.g. rotational kinetic energy). Smaller jumpers actually delivered slightly less mass-specific work during the push-off. This is because they took off with more flexed legs, for reasons explained elsewhere [2]. Nevertheless, smaller jumpers jumped higher because they converted a larger fraction of the work into effective kinetic energy. In other words, smaller jumpers achieved higher take-off velocities because of a higher efficacy.

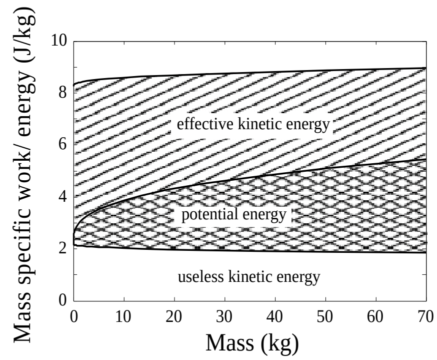

Figure 1: Energy expenditure for different sized jumpers in the absence of air resistance. Total energy expenditure is divided into effective kinetic energy (hatched), potential energy (double hatched) and 'useless' kinetic energy (solid white).

CONCLUSIONS

To conclude, size does matter in jumping. If all animals were geometrically similar and delivered the same amount of work per kg body mass, small jumpers would jump higher than larger ones. In nature, small jumpers do not consistently jump higher than larger ones, hence they are not geometrically similar and it can be predicted that small jumpers require relatively less muscle mass. According to the literature, relative jumping muscle mass amounts to 25-40% of the body weight in the galago, 11-15% in various frogs and only 4-6% in locust. A small jumper needs relatively less muscle mass than a large jumper to achieve a certain take-off velocity. Muscle tissue is energetically expensive for an animal because of its high (resting) metabolism. If the benefits of high take-off velocity (i.e. fast escape from predators) are combined with the challenge to sustain as little muscle tissue as possible, being small seems to present the best compromise.

REFERENCES

1. Alexander, R. M. *Philosophical Transactions of the Royal Society of London. Series B: Biological Sciences* **347**, 235-248 (1995).
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