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A STOCHASTIC MODEL FOR LIGAMENT MECHANICAL FAILURE

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INTRODUCTION

Ligaments are composed of elastin and collagen fibers embedded in a matrix of water, proteoglycans, glycolipids and fibroblasts. Their main function is to guide and to restrain joint motion in order to maintain joint stability. The joint stability is seriously affected when ligament injuries, such as second-degree and third-degree sprains, occur. The treatment and prevention of these injuries require a thorough understanding of the ligament failure mechanisms.

In the present study, a constitutive model will be formulated to describe the failure behavior of ligamentous tissue by taking into account the tissue morphology. The model is able to reproduce the toe region, the linear region and, most importantly, the failure region of the stretch-stress relationship as observed in tensile tests along the fiber direction of the medial collateral ligaments (MCLs) [1].

MODEL FORMULATION

The ligamentous tissue is idealized as composed of N parallel collagen fibers. Elastin and matrix contributions to the tissue mechanical behavior are neglected. The collagen fibers are assumed to contribute to the tissue load only after becoming straight and before breaking. Moreover, they are modeled as a linear elastic material with negligible bending stiffness.

Each fiber is characterized by a straightening stretch, λ^i_s, and a failure stretch, λ^i_f. The fiber straightening stretches and failure stretches for the N fibers are distributed randomly according to Weibull cumulative distributions,

$$G_1(\lambda_s^i) = 1 - e^{\left(\frac{\lambda_s^i - 1}{\beta_s}\right)^{\alpha_s}} \quad \text{and} \quad G_2(\lambda_f^i) = 1 - e^{\left(\frac{\lambda_f^i - 1}{\beta_f}\right)^{\alpha_f}} \quad (i = 1 \ldots N),$$

where α_s, α_f are the shape parameters and β_s, β_f are the scale parameters. The overall tissue stress, σ, and the individual fiber stresses, σ^i, are defined as follows:

$$\sigma = \frac{1}{N}\sum_{i=1}^{N}\sigma^i \quad \text{with} \quad \sigma^i = \begin{cases} K\left(\dfrac{\lambda}{\lambda_s^i} - 1\right) & \text{if } 1 < \dfrac{\lambda}{\lambda_s^i} < \lambda_f^i; \\ 0 & \text{otherwise,} \end{cases}$$

where K is the fiber stiffness, λ is the overall tissue stretch, and λ/λ^i_s is the fiber stretch relative to the taut configuration. Finally, five structural parameters, α_s, α_f, β_s, β_f, K, need to be determined in order to replicate the typical stress-stretch relationship of ligaments.

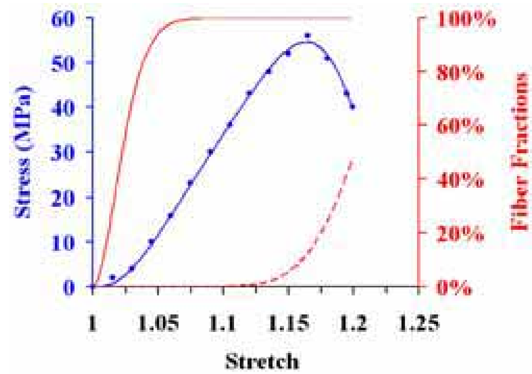

Figure 1: The blue circles are experimental data [1] while the blue line represents the model fit. The red lines represent the straight fiber fraction (—) and the broken fiber fraction (---) predicted by the model.

By following Lanir's approach [2], the presented model is easily generalized to a three-dimensional continuum model to account for the ligament finite deformations and anisotropy.

RESULTS AND DISCUSSION

The model parameter values were determined by curve fitting tensile experimental data on goat MCLs [1]. Their values were found to be K=460 MPa, α_s=1.74, β_s=0.02, α_f=8.10, and β_f=0.18 by implementing the simplex optimization method (R^2=0.99). The experimental data and the model fit at the best fitting parameters are depicted in Figure 1. The fractions of straight fibers and broken fibers, as predicted by the model, are also shown.

The constitutive model is an improvement over the previously presented structural models for connective tissues [3,4] in which the collagen fibers are assumed to have the same failure stretch in the taut configuration.

Three-dimensional constitutive models are necessary to accurately define the ligament mechanical response [5]. However, since quantification of collagen fiber architecture and multiaxial experimental data on ligament failure remain unavailable, the three-dimensional structural model generalization cannot be validated.

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