USINGTHE EIGENVECTOR APPROACH TO LOCATE SPINAL INSTABILITY

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INTRODUCTION

Models have been developed in the past to evaluate stability of the lumbar spine. One approach to quantify the stability index is to determine the smallest eigenvalue associated with a Hessian matrix comprised of all second partial derivatives from a potential energy function [1]. The system is deemed to be unstable whenever the smallest eigenvalue is less than zero. Clinically the location of potential instability is necessary for developing the buttressing motor patterns to prevent instability. Since eigenvectors associated with particular eigenvalues indicate the buckled configurations, this work proposes a particular form of the eigenvector that would enable investigators to determine the location of spinal instability.

METHODS

The lumbar spine model used here includes six vertebral joints (Pelvis/L5-Ribcage/L1) and three axes (flexion/extension, lateral bend, axial rotation) for a total of eighteen degrees of freedom [2]. Each joint is assigned a passive rotational stiffness component. Instability occurs when the stiffness at a joint is compromised along a particular axis. This study compromised the stiffness at various vertebral levels, and particular axes to investigate the plausibility of the proposed eigenvector format.

As a proof of principle, a theoretical trial of a 25 kg load applied to the upright spine was considered. The rotational stiffness parameters [3] for each joint and axis were made to be identical. This forced the rotational stiffness at each joint to be identical. Lastly, we reduced the rotational stiffness at the L3-L4 joint and lateral bend axis to obtain a compromised joint and axis. Thus, the primary instability will occur at the L3-L4 joint and in the lateral bend axis. Subsequently, the known location of the instability can be checked with the proposed form of the eigenvector described below.

RESULTS & DISCUSSION

The Hessian matrix used for the analysis of spinal stability is a symmetric 18 x 18 matrix whose individual entries are second partial derivatives of a potential energy function taken with respect to generalized coordinates at each lumbar joint and axis. The symmetric nature of the Hessian matrix admits the following relationship between the eigenvalues and eigenvectors of the Hessian matrix:

$$VDV^T = H \quad (1)$$

This is where H is the Hessian matrix, D is a diagonal matrix that contains the eigenvalues along its main diagonal, and V is a matrix whose columns are the eigenvectors associated with a particular eigenvalue. V, D, and H are all 18 x 18 matrices. The individual entries of each eigenvector represent a particular vertebral level and axis (Table 1). Consequently, the joint and axis of buckling is indicated by the largest component of the eigenvector associated with the smallest eigenvalue.

The smallest eigenvalue from the example indicated that an instability had occurred ($?_{min}$ = -18.7231). The largest entry of the eigenvector associated with the smallest eigenvalue occurred along the lateral bend axis of the L3-L4 joint (Table 1). This is precisely the joint and axis with compromised rotational stiffness.

Interestingly, the eigenvector also has large components along the lateral bend axis for all joints above L3-L4. These components are all smaller than the component at L3-L4 along the lateral bend axis, and decrease as the segmental level reaches the top of the spine. This suggests that the entries of the eigenvector demonstrate a deflection of the vertebral joint from some reference orientation.

CONCLUSIONS

The example illustrates the plausibility of the proposed form for an eigenvector in the solution of spinal instability when such instability occurs. This interpretation of the eigenvector allows investigators to develop efficient motor patterns that will buttress instability.

REFERENCES

1. Howarth et. al. *J Biomech* **37**, 1147-1154, 2004.
2. Cholewicki et. al. *Clin Biomech* **11**, 1-15, 1996.
3. McGill et. al. *Spine* **19**, 696-704, 1994.

Table 1: Entries of the eigenvector associated with the smallest eigenvalue. The largest value is noted with an asterisk. RC = Ribcage, PELV = Pelvis, FE = Flexion/Extension, LB = Lateral Bend, A R = Axial Rotation.

Joint		**RC-L1**			**L1-L2**			**L2-L3**	
Axis	**FE**	**LB**	**AR**	**FE**	**LB**	**AR**	**FE**	**LB**	**AR**
Entry	0.0395	0.3350	-0.0093	0.0294	0.4568	0.0099	0.0222	0.5527	0.0272

Joint		**L3-L4**			**L4-L5**			**L5-PELV**	
Axis	**FE**	**LB**	**AR**	**FE**	**LB**	**AR**	**FE**	**LB**	**AR**
Entry	0.0161	0.6049*	0.0477	0.0109	0.0073	0.0295	0.0057	0.0037	0.0127