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A NON-LINEAR, ANISOTROPIC, INHOMOGENEOUS MODEL OF ARTICULAR CARTILAGE

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INTRODUCTION

Articular cartilage is a biological composite material made of a proteoglycan matrix, and of chondrocyte and collagen fibre inclusions. We have modelled articular cartilage by means of a linearly elastic Transversely Isotropic, Transversely Homogeneous (TITH) model [1]. Here, we extend the TITH model to non-linear elasticity by using a class of transversely isotropic potentials [2,3]. The use of a non homogeneous potential (i.e., explicitly dependent on the point) enables us to identify non-uniformities in the simulation of compression tests.

METHODS

The stress strain relationship for a hyperelastic material is:

$$\sigma_{ij} = \frac{2}{J} F_{ir} \frac{\partial U}{\partial C_{rs}}(C)\, F_{js} \tag{1}$$

where σ is the Cauchy stress, and U is the strain energy potential which is a function of the right Cauchy stretch, C. For a transversely isotropic material, with transverse plane orthogonal to the unit vector $w = e_1$, U is a function of the three principal invariants of C, and the two additional invariants for transverse isotropy [4]. By expressing the invariants explicitly in terms of the components of C, we have:

$$\begin{aligned} U(C) &= a\ \exp(f(C)) \\ f(C) &= -b\ \ln(\det(C)) + \alpha_1 (C_{rr} - 3) + \\ &\quad + \tfrac{1}{2}\alpha_2((C_{rr})^2 - C_{rs}C_{sr} - 6) + \\ &\quad + \alpha_3 (C_{rr} - 3)(w_r C_{rs} w_s - 1) + \alpha_4 (w_r C_{rs} w_s - 1) + \\ &\quad + \alpha_5 (w_r C_{rh} C_{hs} w_s - 1) + \alpha_6 (w_r C_{rs} w_s - 1)^2 + \alpha_7 (C_{rr} - 3)^2 \end{aligned} \tag{2}$$

The coefficients a, b, $\alpha_1, \ldots, \alpha_7$ are determined by imposing that, for small strains, Eq. (1) must reduce to the linear theory. If we use Cohen's potential [3], then $b = 1$ and $\alpha_7 = 0$. For a confined compression test, the stress strain relationship is:

$$\sigma_{11} = 2a\ (2P\lambda + Q\lambda^{-1} - \lambda^{-3})\ \exp(P\lambda^4 + Q\lambda^2 - R) \tag{3}$$

where λ is the axial Cauchy stretch, and P, Q and R are linear combinations of $\alpha_1, \ldots, \alpha_6$. In the non-linear TITH model, these coefficients explicitly depend on the tissue depth via the non dimensional coordinate, ξ, running from the tidemark ($\xi = 0$) to the articular surface ($\xi = 1$). We plotted the stress-strain curve for several values of ξ (Fig. 1). We assumed that, for a transversely homogeneous material in confined compression, the axial stress must be uniform, and imposed a 0.5 MPa stress (Fig. 1). We then plotted the corresponding values of the strain at each depth ξ (Fig. 2) and obtained the spatial distribution of the Green strain, $\varepsilon = (\lambda^2 - 1)/2$, for the non-linear TITH model, and Cohen's non-linear homogeneous model.

RESULTS AND DISCUSSION

The results of this study show that, while the homogeneous model can only give rise to an intrinsic uniform solution (Fig. 2), the non-linear TITH model is able to predict non uniformities in axial strain, due to the variation of the tangent stiffness with depth: the superficial layers deform more than the deep layers because they are softer, a result that is consistent with experimental evidence [5].

Implementation of this model into a Finite Element code might help in studying the mechanical behaviour of chondrocytes during cartilage deformation, which might proof to be an essential piece towards understanding biological responses of articular cartilage as a function of mechanical loading.

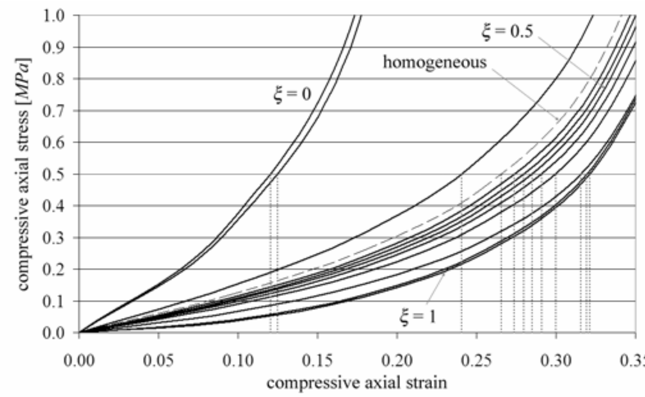

Figure 1: stress-strain curves at various ξ for the non-linear TITH model (solid lines) and Cohen's homogeneous model (dashed line); the dotted lines indicate the strain at each ξ, for a -0.5 MPa stress.

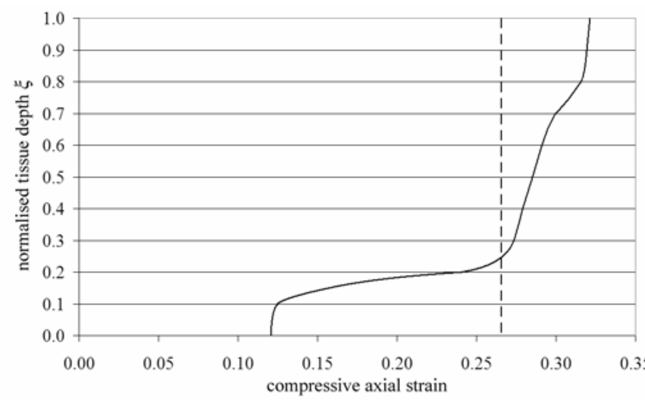

Figure 2: spatial distribution of the strain for the non-linear TITH model (solid line) and Cohen's homogeneous model (dashed line).

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