FORWARD DYNAMICS SOLUTIONS TO MULTI-MODAL INVERSE DYNAMICS PROBLEMS

Behzad Dariush
Honda Research Institute, Mountain View CA USA, bdariush@honda-ri.com

INTRODUCTION

In human motion studies, inverse dynamics analysis is the standard technique used to gain insight into the net summation of all muscle activity at each joint. In the inverse dynamics method, the joint forces and joint moments of force are calculated from a prescribed movement. Since the segmental movements, in contrast to the internal forces, can be measured, this method is commonly applied for the analysis of measured movements.

A full kinematic description obtained from motion capture of marker positions is sufficient to obtain an inverse solution; however, motion capture is often combined with output from other sensors, including force plates, in order to improve the precision of the estimated joint loads. Since the optimal representation of the dynamic equations of motion will differ depending on the available sensors, inverse dynamics is in general considered a multi-modal sensing problem (Dariush et al. 2002). Regardless of the sensing modes, there are two fundamental limitations associated with all inverse dynamics problems. First, the inverse dynamics equations are functions of linear and angular accelerations of body segments, requiring the calculations of higher order derivatives of experimental data contaminated by noise – a notoriously error prone operation (Cullum 1971). Second, the applicability of inverse dynamics is limited to the "analysis" problem. In other words, the solution cannot be used directly to answer the "what if" type questions (or the "synthesis" problem) typically encountered in clinical applications and addressed by forward dynamics simulations.

To address the problem of noise in numerical differntiation, sophisticated numerical schemes are available and provide estimates of higher order derivatives (Hatze 1981, Woltring 1985, Simon 1991); however, the reliability of results is limited since there is no optimal solution or all-purpose automatic method to filter biomechanical data (Giakas and Baltzopoulos 1997). To avoid errors due to higher order derivatives, researchers have combined optimization techniques with a forward dynamics method to calculate the joint moments of force. In particular, Chao and Rim (Chao and Rim 1973) used steepest descent optimization in liu of explicit differentiation for normal walking motion. In their method, convergence and stability are not guaranteed and a solution often requires a very good initial guess of the joint torques. Runge et al. (Runge et al 1995) used a linear quadratic follower (LQF) to estimate the joint moments exerted by humans perturbed by support surface movements. The LQF algorithm does not employ numerical differentiation as constraints, but rather employs constraints based on specification of a cost function. The method produces a stable solution, and is guaranteed to converge. However, the LQF is based on linearization about an operating region and the results have been demonstrated for a small range of motion within this linear operating region. More recently, Dariush et al (Dariush et. al. 2002) demonstrated that the proposed LQF algorithm is robust for a large range of motion, beyond the range of linearization.

This paper presents a new control theoretic framework for the analysis and synthesis of human motion whereby the estimation of internal forces and moments has been formulated as a trajectory tracking control problem. The objective of the tracking controller is to obtain a set of forces and moments that when applied to a forward model will reproduce or track the measured kinematic data. In particular, tracking is achieved by employing a non-linear control law that linearizes and decouples the states of the system. This technique represents a forward dynamics solution to the general multi-modal inverse dynamics problem. The proposed algorithm overcomes the limitations of the aforementioned methods and represents a simple, yet powerful method for both the analysis and synthesis of human motion. In particular, the proposed method is stable, guaranteed to converge, computationally efficient (real time), does not require acceleration computations, and is predictive.

METHODS

The proposed approach uses the tandem combination of inverse and forward dynamics with the trajectory tracking feedback structure shown in Figure 1. The required sensory inputs include the measured kinematics derived from motion capture of markers attached to the body. Additional sensing modes, such as ground reaction forces, can be optionally incorporated to improve the precision. The tracking controller estimates a set of joint loads that when applied to the forward dynamics integration block, would generate simulated kinematics that are identical to the measured kinematics.

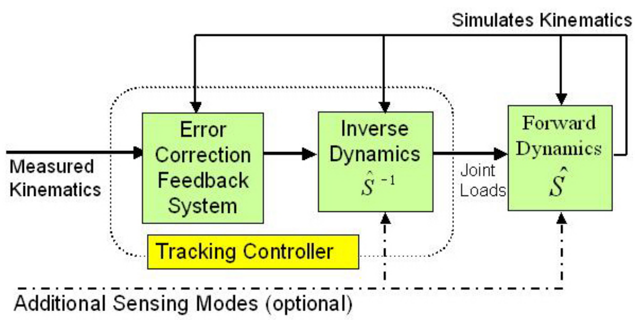

Figure 1: Architecture for the proposed joint load estimation method.

The representation of the equations in the inverse and forward dynamics depends on the available sensors. For a given sensing modality, there corresponds a set of forward dynamics equations S that describe the motion of the musculoskeletal system. Since S is never known precisely, we can estimate (or model) S and represent it by $\hat{S}$. The inverse dynamics model, $\hat{S}^{-1}$, is a transformation of quantities obtained from a set of inputs derived from measurements to the estimated joint loads.

Let q_d represent a vector of “desired” generalized coordinates derived from kinematic measurements, i.e motion capture data. The vectors $\dot{q}_d$ and $\ddot{q}_d$ represent the velocity and acceleration of the generalized coordinates, respectively. Let the vector P represent measurements from additional sensing modalities, which may be optionally used in the inverse dynamics analysis. The optional measurements may include, for example, force plate data, or measurements from accelerometers.

In inverse dynamics analysis, the joint loads, denoted here by the vector *U*, are computed as an algebraic function of the kinematics, their first and second derivatives, and other sensory inputs.

$$U = \hat{S}^{-1}(q_d, \dot{q}_d, \ddot{q}_d, P) \quad (1)$$

In the proposed method, the solution is based on a control law that tracks the desired (or measured) kinematics. The proposed forward dynamics solution to the multi-modal inverse dynamics problem has the following form,

$$U = \hat{S}^{-1}(q, \dot{q}, \ddot{q}^*, P) \quad (2)$$

where

$$\ddot{q}^* = \ddot{q}_d + K_v(\dot{q}_d - \dot{q}) + K_P(q_d - q) \quad (3)$$

The control law in Equation 2 computes the joint loads, U, that when applied to $\hat{S}$ would generate the simulated motion q and reproduce the desired kinematics q_d. The tracking error $e = q_d - q$ can be forced to zero with the fastest possible non-oscillatory response by increasing the position feedback gains K_p and constraining the velocity feedback gains K_v to produce a critically damped response (Craig, 1989).

$$K_v = 2\sqrt{K_P} \quad (4)$$

The detailed block-diagram of the proposed method is illustrated in Figure 2.

As in the inverse dynamics approach, the inputs in the proposed algorithm include the desired kinematics, q_d, derived from motion capture measurements, and estimates of their velocities, $\dot{q}_d$. If additional sensory inputs are available, such as ground reaction forces, they may be incorporated in the vector P. A major distinction of the proposed formulation over the inverse dynamics approach is that the method incorporates feedback from simulated states, and is therefore a forward dynamics solution. As will be shown in the next section, the tracking error can be forced to zero by adjusting the feedback gains, regardless of whether or not the accelerations, $\ddot{q}_d$, are used. Therefore, the desired acceleration term should be included only if there is high confidence in the reliability of the data and the filtering method. Here, the term $\ddot{q}^*$ is referred to as the error corrected acceleration term. It involves position and velocity feedback which completely linearizes and decouples all the states. In other words, the method is guaranteed to achieve near perfect tracking for each state, independently and in real time. In the following section, the error dynamics of the proposed algorithm are presented in detail.

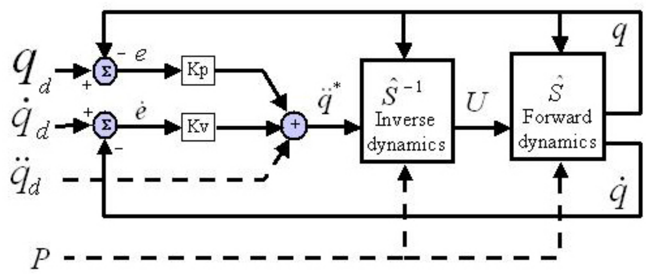

Figure 2: Detailed block-diagram of the proposed method. The inputs include the measured (or desired) kinematics described by a vector of generalized coordinates *q*, and optional vector of additional sensory inputs *P*. The dashed lines are optional.

Error Dynamics

The joint torque and force estimation problem has been formulated as a tracking problem using the nonlinear feedback control law described by Equation 2. In order to analyze the tracking performance, it is instructive to consider the closed loop error dynamics. Consider the output of the inverse model in Figure 2, representing the control law in Equation 2. If this control law is applied to the forward model, the following closed loop relation is obtained,

$$\ddot{q}_d - \ddot{q} + K_v(\dot{q}_d - \dot{q}) + K_P(q_d - q) \quad (5)$$

Let e denote the error between the measured kinematics, q_d, and the simulated state variable, q, obtained by integration in the direct model.

$$e = q_d - q \quad (6)$$

In the following, the error dynamics for two scenarios are considered.

Case 1: Accelerations are included:

If accelerations are included, and in the ideal situation of perfect measurements and zero error in numerical differentiation, the closed loop error dynamics is given by,

$$\ddot{e} + K_v\dot{e} + K_P = 0 \quad (7)$$

The error for each state can be independently controlled by eigenvalue assignment. Let λ_1 and λ_2 denote the eigenvalues of the above differential equation. A critically damped solution with real and equal eigenvalues is given by

$$e(t) = c_1 e^{\lambda_1 t} + c_2\, t\, e^{\lambda_2 t} = 0 \quad (8)$$

It is obvious that the error, as given by Equation 8, can be forced to zero if the eignevalues are negative and large.

The relation between K_v and K_P to achieve a critically damped response is given by

$$K_v = 2\sqrt{K_P} \quad (9)$$

A critically damped solution is desired because it yields the fastest possible non-oscillatory response. Selecting a positive and large feedback gain will force the error to zero.

Case 2: Ignoring accelerations:

Suppose the desired accelerations estimated from double differentiation of measured kinematics are ignored by neglecting the term $\ddot{q}_d$. The closed loop error dynamics of this system is expressed by the following non-homogeneous differential equation.

$$\ddot{e} + K_v\dot{e} + K_P = \ddot{q}_d \quad (10)$$

Although the solution to the above differential equation contains a forcing term, assuming the acceleration term $\ddot{q}_d$ is bounded, the error will converge to zero by assigning the eigenvalues to have negative and real parts. As before, the feedback gains may be appropriately designed for a critically damped response by using Equation 9.

Illustrative Example

A typical application of inverse dynamics analysis involves integration of motion capture and force plate measurements as inputs to an iterative "ground up" Newton-Euler inverse dynamics procedure. This procedure is referred to as the traditional inverse dynamics problem. The proposed concept. which represents a forward dynamics solution to the traditional inverse dynamics problem is achievable using the principles developed in this paper. The detailed analysis and formulation of the proposed method for the combined motion capture and force plate sensing modality involves three steps as outlined below.

The first step involves formulating the dynamic equations of motion as a recursive ground up inverse dynamics problem. To do so, consider the free body diagram of an isolated body segment which forms a serial chain with its two neighboring segments (see Figure 3). This figure illustrates the description of the global frame, position vectors, and all forces and moments acting on the body.

Suppose body segment i is connected in a serial chain to its neighbors, body segment i-1 and body segment i+1. Let X and θ be the generalized coordinates, representing the position of the center of mass and the angle with respect to the y-axis, respectively. The vectors $L_P = [L_{PX} \quad L_{Py}]^T$, and $L_D = [L_{DX} \quad L_{Dy}]^T$ originate at the center of mass and terminate at the

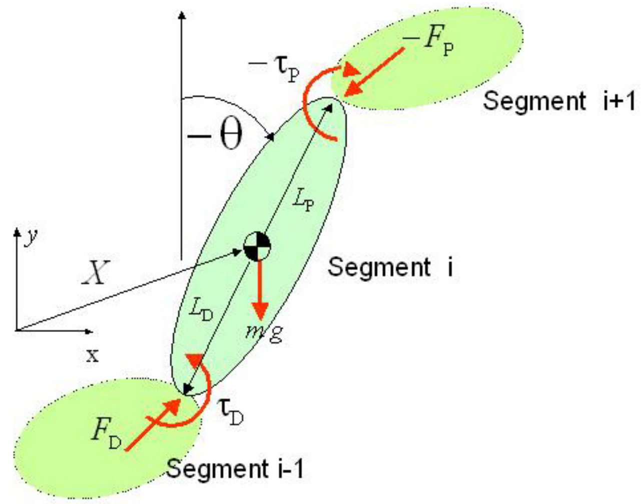

Figure 3: Free body diagram of an isolated body segment i attached in a serial chain to it's neighbours i-1 and i+1.

proximal and distal joint positions, respectively. Let $\ddot{X}$, $\ddot{\theta}$ be respectively, the accelerations of the center of mass, and angular acceleration. Let m be the mass, I the moment of inertia, and g the acceleration due to gravity. Let the joint torque and reaction force at the proximal and distal segments be described by τ_D, τ_P, F_D, and F_P, respectively. The Newton Euler equations for an isolated body segment is given by,

$$m\ddot{X} = F_D - F_P - \mathrm{m\,g} \quad (11)$$

$$I\ddot{\theta} = L_D \times F_D - L_P \times F_P + \tau_D - \tau_P \quad (12)$$

In step 2 of the proposed algorithm, the Newton Euler equations are expressed in matrix form in such a way that all generalized coordinates, inputs, and outputs, are separated by linear matrix algebra.

$$M\ddot{q} = A_P(q)U_P + A_D(q)U_D + P \quad (13)$$

where the generalized coordinates, are represented by the 3x1 vector q

$$q = [X \quad \theta] \quad (14)$$

The joint loads at the proximal and distal section are represented by a 3x1 vector, respectively

$$U_D = [F_D \quad \tau_D]^T \quad (15)$$

$$U_P = [F_P \quad \tau_P]^T \quad (16)$$

The mass matrix is given by the 3x3 matrix ,

$$M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{bmatrix} \quad (17)$$

The 3x1 vector P describes the gravitational forces

$$P = [0 \quad -mg \quad 0]^T \tag{18}$$

And the 3x3 matrices A_D and A_P are functions of the generalized coordinate,

$$A_D(q) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -L_{DY} & L_{DX} & 1 \end{bmatrix} \tag{19}$$

$$A_P(q) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ L_{PY} & -L_{PX} & -1 \end{bmatrix} \tag{20}$$

By substituting the desired generalized coordinates , q_d , for q and rearranging terms in Equation 13, the proximal joint loads can be expressed in terms of the distal joint loads using the following relation,

$$U_P = A_P(q_d)^{-1}(M\,\ddot{q}_d - A_D(q_d)U_D - P) \tag{21}$$

Equation 21 represents the standard inverse dynamics relation for an iterative 2D analysis of an isolated body segment. This procedure can be repeated iteratively, using the ground reaction force as a constraint to recursively compute the forces and moments at the proximal end of each successive segment.

In step three of the proposed algorithm, the input to the inverse dynamics procedure given by Equation 21 involves a feedback term. The solution is expressed by

$$U_P = A_P(q)^{-1}(M\,\ddot{q}^* - A_D(q)U_D - P) \tag{22}$$

where $\ddot{q}^*$ is given by Equation 3. The distinction between the proposed solution given by Equation 22, and the standard inverse dynamics solution given by Equation 21 is as follows:

- The desired accelerations $\ddot{q}_d$ have been replaced with the error correction acceleration, $\ddot{q}^*$,which does not explicitly require the second order derivative of the measured generalized coordinates.
- The matrices $A_P(q_d)$ and $A_D(q_d)$, which are functions of the desired generalized coordinates, have been are replaced with $A_P(q)$ and $A_D(q)$ which are functions of the simulated generalized coordinates.

RESULTS AND DISCUSSION

Synthetically generated kinematic data for rhythmic motion was injected with random noise of variance 5 mm and used as input to drive a three link lower extremity planar model. Figure 4 illustrates the mean absolute tracking error $|e|$ of the hip flexion angle and the mean error of the estimated hip joint torque (normalized by the range in torque values) as a function of K_p. The results support the theory that by increasing the feedback gain, the tracking error can be forced to nearly zero. This trend is evidenced for a number of other simulations, including ones with real experimental data. Conceptually, if the tracking error can be forced to zero, then the torque error can be forced to zero as well. Note that in these simulations, accelerations obtained from the desired (or measured) kinematics were not included in the control law.

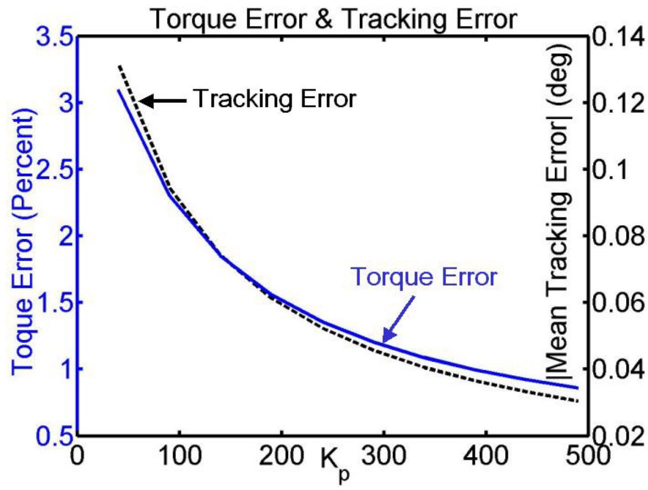

Figure 4: Tracking error and Torque estimation error as a function of the feedback gain Kp.

In the proposed formulation, the parameters of the forward dynamic model can be changed in order to observe the predictive capability of the method. The type of question that can be posed may be stated as "what would happen if….". For example, an orthopedic surgeon may ask "what would happen if I transferred a muscle from one insertion point to another?" Or a biomechanics researcher may ask "what would happen if I changed the physical parameters of the forward model….how would that alter the resulting motion?" Such phenomenon can be observed because the analysis is based on forward dynamic simulation. If the forward dynamics model differs from the inverse dynamics model, the simulated motion will not track the measured motion. This property opens the door to addressing a wide range of "what if" type questions.

SUMMARY

A new algorithm has been presented for analysis and synthesis of human motion from different sensing modalities. The proposed algorithm has the following unique attributes:

- It is shown that the tracking error can be forced to zero without using acceleration estimates from noisy marker measurements.
- The solution to obtain estimates of joint load is fast (real time), guaranteed to converge, and is self starting, i.e. does not require an initial solution.
- The method is very general and can be applied to any class of multi-sensory joint torque estimation problem.
- The method is based on a forward dynamics solution and is therefore capable of predicting new motions by altering $\hat{S}$, a very useful attribute for answering "what if" type questions.

REFERENCES

Chao, E.Y., Rim, K. (1973) Application of optimization principles in determining the applied moments in human leg joints during gait. *J. Biomech*, **6**, 497-510.

Cullum, J., (1971) Numerical differentiation and regularization, *SIAM J. Numer. Anal*. **8**(2), 254-265.

Craig J. (1989) *Introduction to Robotics*. Addison Wesley.

Dariush B. et al. (2002) Multi-modal analysis of human movement from external measurements. *J. Dyn. Sys. Meas & Control.* **123**(2), 272-278.

Giakas G., Baltzopoulos, V. (1997) Optimal digital filtering requires a different cut-off frequency strategy for the determination of the higher derivatives. *J. Biomech*, **30**(8). 851-855.

Hatze (1981) The use of optimally regularized Fourier series for estimating higher-order derivatives of noisy biomechanical data. *J. Biomech*. **14**, 13-18.

Runge, C.F. et al. (1995) Estimating net joint torques from kinesiological data using optimal linear system theory *IEEE Trans. BME*, **42**(12), 1158-1164.

Woltring H. (1985) On optimal smoothing and derivative estimation from noisy displacement data in biomechanics. *Hum Movement Sci*, **4**, 229-245